Relative Value Concepts

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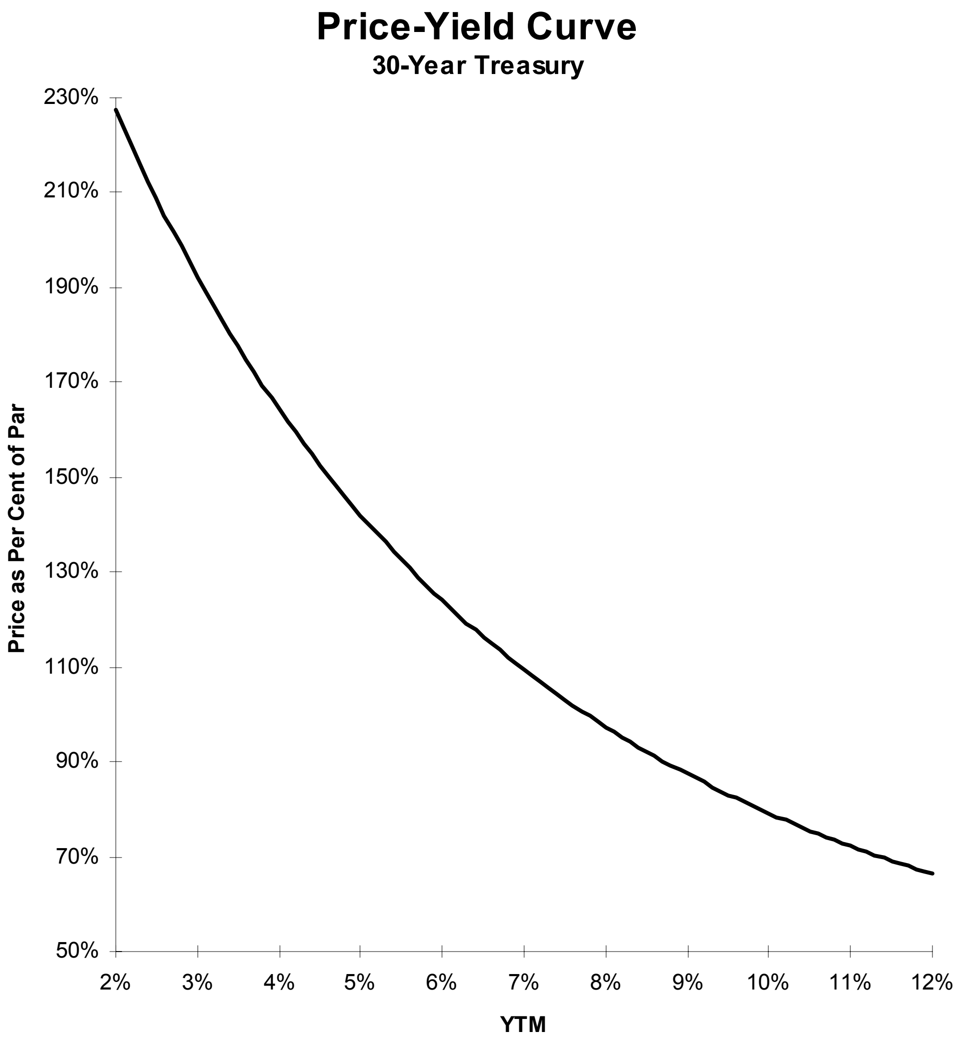
DURATION AND CONVEXITY

Below is a picture of the price-yield relationship for the U.S. Treasury long bond in early May 1995:



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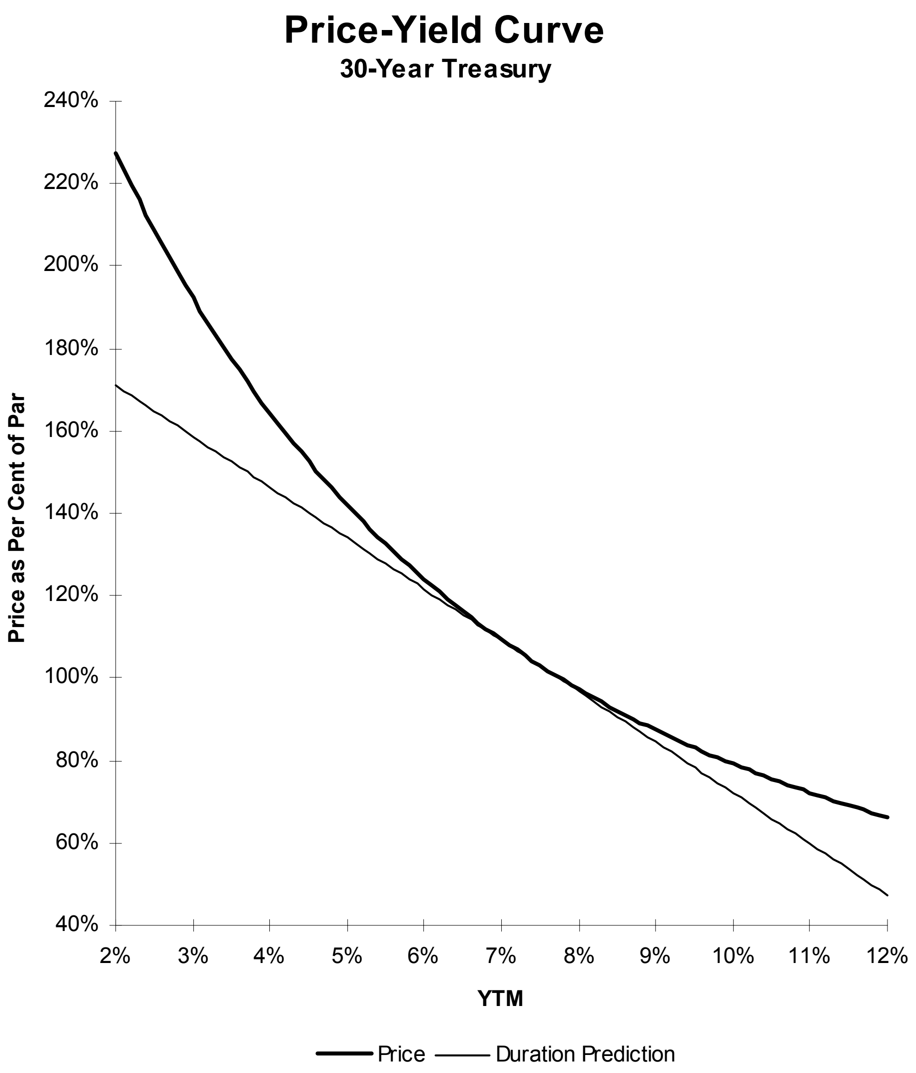
This bond has the following characteristics:

Settlement:	2-May-95
Issue:	15-Feb-95
Maturity:	15-Feb-25
Coupon:	7 5/8 %
Price:	103 11/32
Accrued:	1.6008
Dirty Price:	104.9446
YTM:	7.3456%

Modified duration does a reasonable job of describing price changes for small changes in the bond's YTM. For larger changes, however, it does not do a very good job:



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Interestingly, modified duration is "wrong" in the same way whether rates move up or down. Its price prediction is **always below the true price**.

Risk Structure of Interest Rates

Bonds with greater duration have greater price risk. The yield curve compares yields on bonds with different maturities in order to obtain information about yield and maturity.

It is therefore reasonable to ask if there is a relationship between yield and price risk, or, more specifically, duration. The table below compares maturities and YTM for U.S. Treasuries having approximately 20 years remaining maturity in early May 1995.

Maturity	Coupon	Price	YTM	Modified Duration
15-Feb-15	11 1/4 %	139 10/32	7.4255%	9.3628
15-Aug-15	10 5/8 %	133 4/32	7.4351%	9.5637
15-Nov-15	9 7/8 %	125 13/32	7.4412%	9.5491
15-Feb-16	9 1/4 %	118 27/32	7.4514%	9.9062
15-May-16	7 1/4 %	97 29/32	7.4482%	10.2487
15-Nov-16	7 1/2 %	100 18/32	7.4467%	10.2782
15-May-17	8 3/4 %	113 23/32	7.4706%	10.0543

We can look at this particular portion of the yield curve graphically in the usual way:

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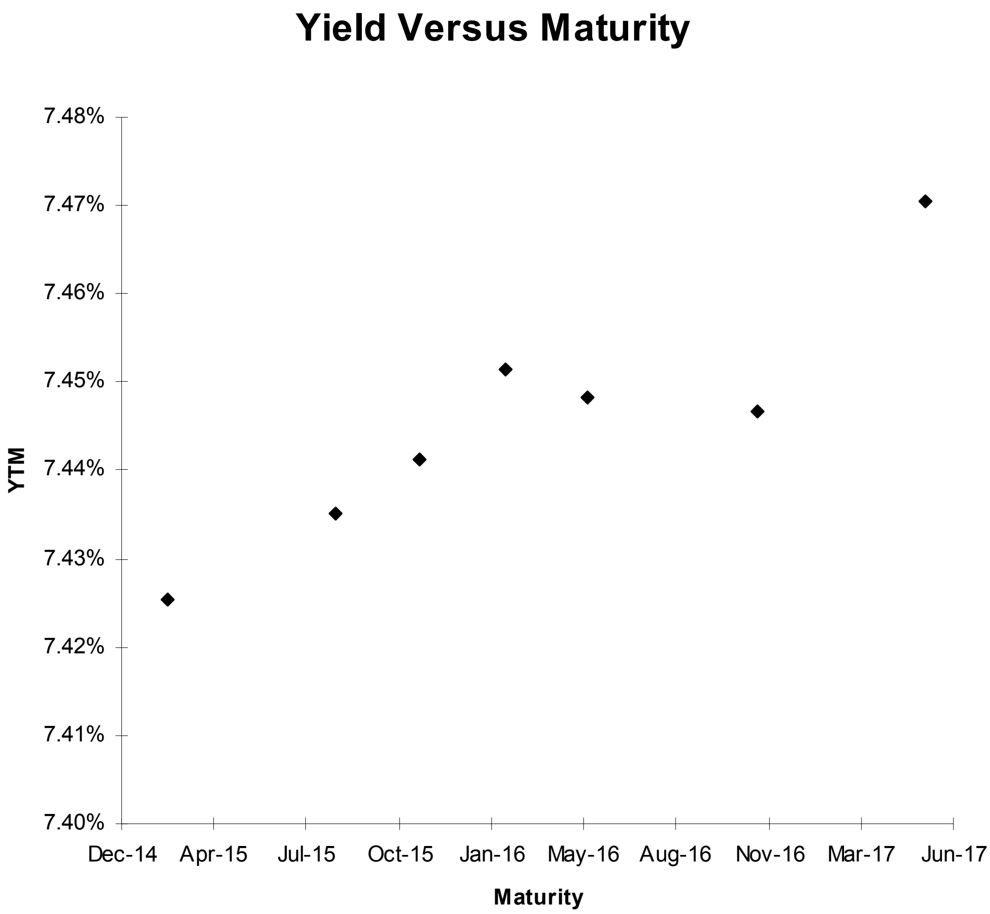

The yield on the bond maturing 15 February 2016 looks about a basis point too high, but otherwise the curve is fairly ordinary-looking.

We can also compare yield with duration:

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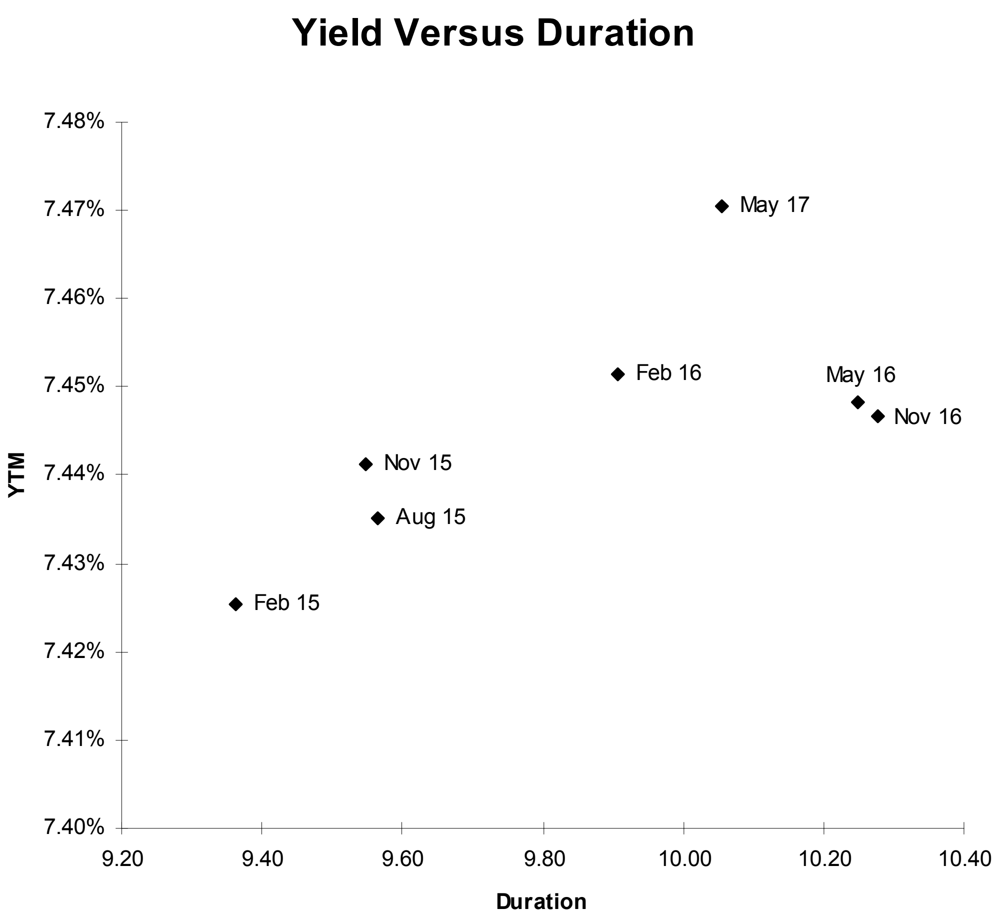

Because its coupon is some 1.25% higher than the bonds maturing in May and November 2016, the May 2017 bond has a **lower** duration. By trading it at a slightly higher yield, the market would seem to be underpricing it slightly, given its lower risk.

Bond Swaps

Bond swaps are sometimes structured based on this logic. for example, an investor might wish to increase yield while decreasing risk. In that case he could:

Action	Maturity	Coupon	Price	YTM	Modified Duration
Sell	15-Nov-16	7 1/2 %	100 18/32	7.4467%	10.2782
Buy	15-May-17	8 3/4 %	113 23/32	7.4706%	10.0543

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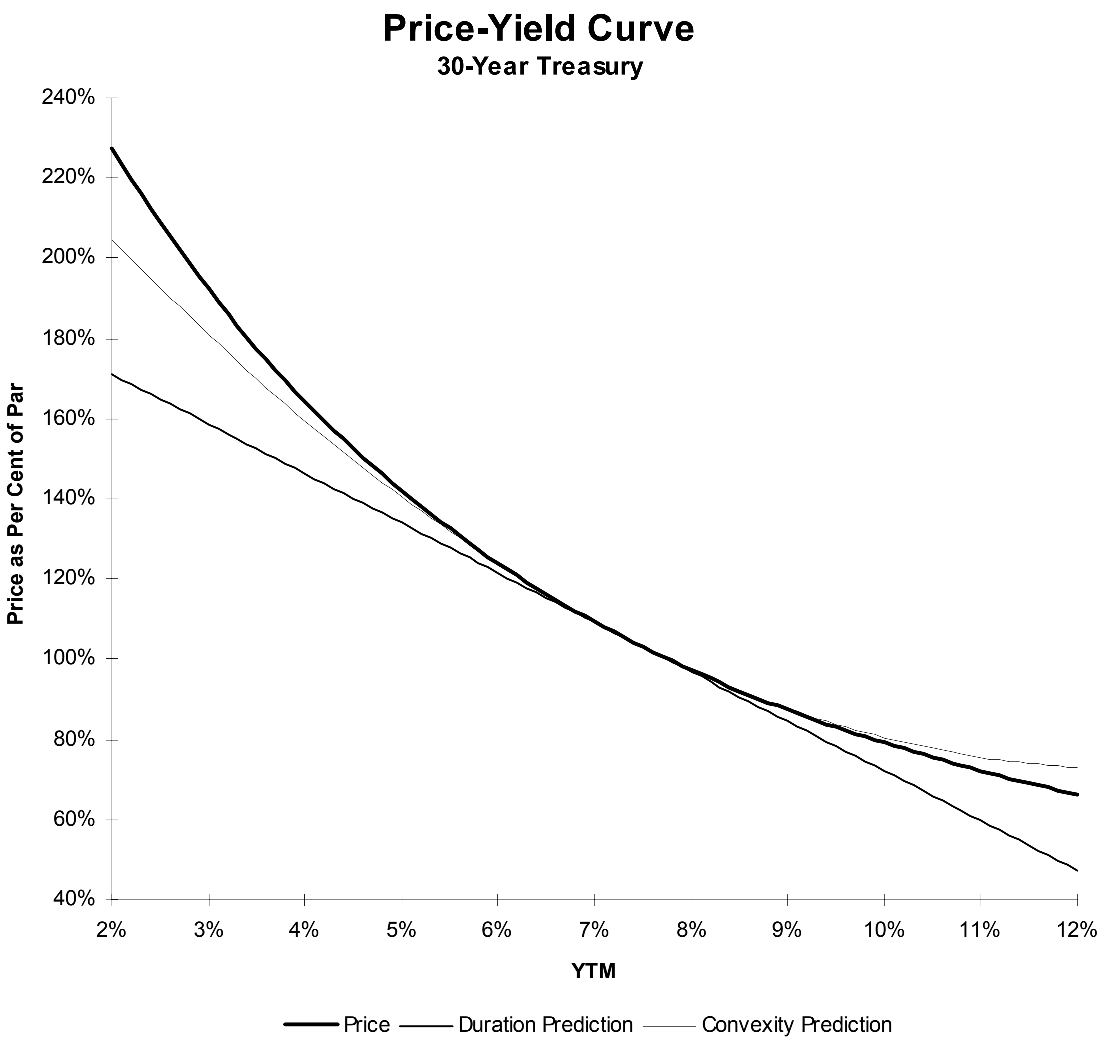
Approximating the "True" Price-Yield Curve
In the graph above, we saw that duration does a good job of predicting a bond's price only for relatively small changes in yield.

To better approximate the price for larger changes in yield we need to use convexity:



As can be seen in the graph above, convexity is a desirable quality **for the owner of a bond**. The greater the convexity, the better the bond's performance when yields move lower.

The greater the convexity, the less risk of price drops when yields move higher.

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DUMBBELL-BULLET ANALYSIS

A trading strategy employing views on the shape of the yield curve might be to trade two bonds — one short in maturity and the other long — against a third bond of medium maturity.

The combination of the two bonds is called a ***dumbbell*** (the name for a piece of weightlifting equipment consisting of a hand-held weight with a disk-shaped weight on each end).

The single bond is called a ***bullet***.

The goal of this trade is to create arbitrage profits without taking any risk. We are thus looking to take a long position in one and a short position in the other such that the two positions have equal duration risk and market values.

To illustrate this trading strategy, let us use the following three bonds (value date is 2 May 1995):

Bond	Maturity	Coupon	Price	Accrued	Dirty Price	YTM	Dollar Duration
A	15-Feb-00	7 1/8 %	100 29/32	1.4959	102.4021	6.8960%	4.0451
B	15-Feb-15	11 1/4 %	139 10/32	2.3619	141.6744	7.4255%	13.2629
C	15-Feb-05	7 1/2 %	102 31/32	1.5746	104.5433	7.0724%	7.1131

Bond A is a 5-year Treasury bond.

Bond B is a 20-year Treasury bond, one of those examined above.

Bond C is a 10-year Treasury bond.

The dumbbell will consist of a position in bonds A and B.

The bullet will be a position in C.

Dollar duration is calculated by multiplying the modified duration times the bond's dirty price. It represents the amount of absolute price change the bond will have for a 1% change in interest rates.

This can be shown by using Bond A.

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Duration is calculated as follows. Each cash flow is priced, and then multiplied by time. In the exhibit below, PV factors are based on the bond's YTM, calculated using the A/A convention. Time is based on A/A days.

Date	Cash Flow	PV Factor	CF PV	Time	PV × T
2-May-95			102.3983%		4.091129
15-Aug-95	3.5625%	0.980390	3.4926%	0.287671	0.010047
15-Feb-96	3.5625%	0.947845	3.3767%	0.791781	0.026736
15-Aug-96	3.5625%	0.916253	3.2642%	1.290411	0.042121
17-Feb-97	3.5625%	0.885382	3.1542%	1.800000	0.056775
15-Aug-97	3.5625%	0.856193	3.0502%	2.290411	0.069862
16-Feb-98	3.5625%	0.827500	2.9480%	2.797260	0.082462
17-Aug-98	3.5625%	0.799769	2.8492%	3.295890	0.093906
15-Feb-99	3.5625%	0.773402	2.7552%	3.794521	0.104548
16-Aug-99	3.5625%	0.747484	2.6629%	4.293151	0.114323
15-Feb-00	103.5625%	0.722705	74.8452%	4.794521	3.588467

Duration is the sum of the cash flow PVs times time, divided by the bond's dirty price.

Modified duration is duration divided by 1 plus YTM:

$$\text{Modified Duration} = \frac{4.091129}{\left(1 + \frac{6.8960\%}{2}\right)} = 3.95$$

Modified duration times the bond's dirty price gives us the dollar duration:

$$\text{Dollar Duration} = 3.95 \times 102.3983\% = 4.05$$

Dollar duration can be interpreted as the absolute amount of price change resulting from a 1% change in the bond's YTM.

We need to take a position in Bonds A and B which has the same market value as Bond C. That way, we can finance the trade.

At the same time we want the dollar duration of both positions to be the same. For small movements in interest rates, our two positions will have the same risk.

The first condition might be described as follows:

$$Z \times \left[X \times MV_A + (1 - X) \times MV_B\right] = MV_C$$

where:

Z = The percent of face value of A and B needed to equal C
X = The percent of A needed
1–X = The percent of B needed
MV = The market value of each asset

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The second condition can be described similarly as follows:

$$Z \times \left[X \times DD_A + (1 - X) \times DD_B\right] = DD_C$$

where:

- Z = The percent of face value of A and B needed to equal C
- X = The percent of A needed
- 1–X = The percent of B needed
- DD = The dollar duration of each asset

Using algebra, we can "solve" these two equations simultaneously to determine the exact amounts of A and B needed to exactly offset both the market value and dollar duration of C. Doing so we obtain the following equation for Z:

$$Z = \frac{DD_C - MV_C \times R}{DD_B - MV_B \times R}$$

$$\text{where: } R = \frac{DD_A - DD_B}{MV_A - MV_B}$$

The percent of Bond A can then be calculated as follows:

$$X = \frac{DD_B \times Z - DD_C}{DD_B \times Z - DD_A \times Z}$$

Using the above equations we can solve for the following amounts of A and B:

X	55.3570%
1–X	44.6430%
Z	87.1671%

The market value of the dumbbell is equal to the market value of Bond C:

$$87.1671\% \times \left[55.3570\% \times 102.4021 + 44.6430\% \times 141.6744\right] = 104.5433 = MV_C$$

The dollar duration of the dumbbell is also equal to that of Bond C:

$$87.1671\% \times \left[55.3570\% \times 4.0451 + 44.6430\% \times 13.2629\right] = 7.1131 = DD_C$$

The yield of the dumbbell can be calculated several different ways: weighted average yield, cash flow yield or horizon yield. For now we use the simple weighted average yield.

This is calculated using the respective market value percentages as weights:

$$55.3570\% \times 6.8960 + 44.6430\% \times 7.4255 = 7.1324\%$$

We can now describe our two possible positions more completely:

Bond	Maturity	Coupon	Price	Accrued	Dirty Price	YTM	Dollar Duration

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C	15-Feb-05	7 1/2 %	102 31/32	1.5746	104.5433	7.0724%	7.1139
Dumbbell							
A	15-Feb-00	7 1/8 %	100 29/32	1.4959	49.4122	6.8960%	1.9521
B	15-Feb-15	11 1/4 %	139 10/32	2.3619	55.1311	7.4255%	5.1618
Total					104.5433	7.1324%	7.1139

Arbitrage Result Number 1

This yields a not-very-interesting result: It appears that we can grab a yield gain of only 0.06% by buying the Dumbbell and selling Bond C. The transaction is self-financing and of identical dollar duration. Intuitively, it seems reasonable that the yields should be about the same.

But is the yield difference calculated correctly?

Relative Value

In addition to weighted average yield, we can also compare the yields on the dumbbell and the bullet two other ways: cash flow yield and horizon yield.

Both of these are "better" ways of looking at the relative value of each strategy.

Cash Flow Yield

Calculating the simple average of the yields to maturity of the constituent bonds is a very rough approximation of the dumbbell's actual YTM.

This is because the two bonds have very different maturities.

YTM should be understood to be the single cash flow discounting rate which, when used to price each of the cash flows in the dumbbell, returns their aggregate market value.

There is no easy way to solve for this YTM. The easiest approach is to set it up on a spreadsheet and find the yield value through a solver function.

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Solving for Cash Flow YTM Iteratively

The cash flows in the dumbbell are as follows:

Date	Cash Flow	PV Factor	CF PV	Time
2-May-95		1.000000	104.5433%	
15-Aug-95	3.9079%	0.979318	3.8271%	0.287671
15-Feb-96	3.9079%	0.945064	3.6932%	0.791781
15-Aug-96	3.9079%	0.911867	3.5635%	1.290411
17-Feb-97	3.9079%	0.879489	3.4370%	1.800000
15-Aug-97	3.9079%	0.848930	3.3176%	2.290411
16-Feb-98	3.9079%	0.818948	3.2004%	2.797260
17-Aug-98	3.9079%	0.790025	3.0874%	3.295890
15-Feb-99	3.9079%	0.762576	2.9801%	3.794521
16-Aug-99	3.9079%	0.735643	2.8748%	4.293151
15-Feb-00	52.1611%	0.709943	37.0314%	4.794521
15-Aug-00	2.1889%	0.685005	1.4994%	5.293151
15-Feb-01	2.1889%	0.660943	1.4467%	5.797260
15-Aug-01	2.1889%	0.637726	1.3959%	6.293151
15-Feb-02	2.1889%	0.615325	1.3469%	6.797260
15-Aug-02	2.1889%	0.593711	1.2996%	7.293151
17-Feb-03	2.1889%	0.572629	1.2534%	7.802740
15-Aug-03	2.1889%	0.552733	1.2099%	8.293151
16-Feb-04	2.1889%	0.533212	1.1672%	8.800000
16-Aug-04	2.1889%	0.514482	1.1262%	9.298630
15-Feb-05	2.1889%	0.496508	1.0868%	9.800000
15-Aug-05	2.1889%	0.479067	1.0486%	10.295890
15-Feb-06	2.1889%	0.462239	1.0118%	10.800000
15-Aug-06	2.1889%	0.446002	0.9763%	11.295890
15-Feb-07	2.1889%	0.430336	0.9420%	11.800000
15-Aug-07	2.1889%	0.415219	0.9089%	12.295890
15-Feb-08	2.1889%	0.400634	0.8770%	12.800000
15-Aug-08	2.1889%	0.386561	0.8461%	13.298630
16-Feb-09	2.1889%	0.372909	0.8163%	13.805479
17-Aug-09	2.1889%	0.359739	0.7874%	14.304110
15-Feb-10	2.1889%	0.347239	0.7601%	14.802740
16-Aug-10	2.1889%	0.334976	0.7332%	15.301370
15-Feb-11	2.1889%	0.323273	0.7076%	15.802740
15-Aug-11	2.1889%	0.311917	0.6828%	16.298630
15-Feb-12	2.1889%	0.300961	0.6588%	16.802740
15-Aug-12	2.1889%	0.290389	0.6356%	17.301370
15-Feb-13	2.1889%	0.280189	0.6133%	17.805479
15-Aug-13	2.1889%	0.270346	0.5918%	18.301370
17-Feb-14	2.1889%	0.260747	0.5708%	18.810959
15-Aug-14	2.1889%	0.251687	0.5509%	19.301370
16-Feb-15	41.1029%	0.242798	9.9797%	19.808219

3.9079% represents the total coupon of the dumbbell while both bonds are still outstanding:

$$87.1671\% \times \left(\frac{7.125\%}{2} \times 55.3570\% + \frac{11.125\%}{2} \times 44.6430\% \right) = 3.9079\%$$

Likewise, 2.1889% is the cash flow of the coupon on the longer Bond B.

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The PV factors are based on the YTM we solve for. Each PV factor is calculated using YTM in the following relationship:

$$PVf_n = \frac{1}{\left[1+\frac{YTM}{2}\right]^{\left(Periods-1+\frac{\text{Accrual Days}}{\text{Actual Days in Accrual Period}}\right)}}$$

"Accrual days" refers to the number of days between settlement and the next semi-annual coupon date. "Actual days in accrual period" refers to the number of actual days in the 6-month period ending at the next semi-annual coupon date.

The solver function in Excel returns the following cash flow YTM: 7.2811%

This is the only value for cash flow YTM which yields the correct dumbbell PV of 104.5433%.

Approximate Cash Flow YTM
It is possible to closely approximate the cash flow yield in a portfolio by using the **dollar-duration-weighted average yield** instead of the simple weighted average yield.

The result is pretty good:

$$\frac{6.8960\% \times 1.9521 + 7.4255\% \times 5.1618}{1.9521 + 5.1618} = 7.2802\%$$

We can now describe our two possible positions more accurately:

Bond	Maturity	Coupon	Price	Accrued	Dirty Price	YTM	Dollar Duration
C	15-Feb-05	7 1/2 %	102 31/32	1.5746	104.5433	7.0724%	7.1139
Dumbbell							
A	15-Feb-00	7 1/8 %	100 29/32	1.4959	49.4122	6.8960%	1.9521
B	15-Feb-15	11 1/4 %	139 10/32	2.3619	55.1311	7.4255%	5.1618
Total					104.5433	7.2802%	7.1139

Arbitrage Result Number 2
This result is far more interesting: It appears that we can grab a yield gain of 0.21% by buying the Dumbbell and selling Bond C.

Is there any explanation for this? Does the relative riskiness of the dumbbell versus the bullet give us an explanation?

Relative Riskiness
To measure the relative riskiness of the dumbbell versus the bullet, we need to compute the **convexity** of both.

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Dumbbell Convexity
Convexity is calculated as follows:

Dumbbell	YTM:	7.2811%			Convexity:	84.934235
Date	Cash Flow	PV Factor	CF PV	Time	PV × T	PV × T²
2-May-95		1.000000	104.5433%			
15-Aug-95	3.9079%	0.979318	3.8271%	0.287671	0.011009	0.003167
15-Feb-96	3.9079%	0.945064	3.6932%	0.791781	0.029242	0.023154
15-Aug-96	3.9079%	0.911867	3.5635%	1.290411	0.045984	0.059338
17-Feb-97	3.9079%	0.879489	3.4370%	1.800000	0.061866	0.111358
15-Aug-97	3.9079%	0.848930	3.3176%	2.290411	0.075986	0.174039
16-Feb-98	3.9079%	0.818948	3.2004%	2.797260	0.089523	0.250420
17-Aug-98	3.9079%	0.790025	3.0874%	3.295890	0.101756	0.335377
15-Feb-99	3.9079%	0.762576	2.9801%	3.794521	0.113080	0.429085
16-Aug-99	3.9079%	0.735643	2.8748%	4.293151	0.123421	0.529866
15-Feb-00	52.1611%	0.709943	37.0314%	4.794521	1.775477	8.512561
15-Aug-00	2.1889%	0.685005	1.4994%	5.293151	0.079366	0.420098
15-Feb-01	2.1889%	0.660943	1.4467%	5.797260	0.083872	0.486225
15-Aug-01	2.1889%	0.637726	1.3959%	6.293151	0.087848	0.552839
15-Feb-02	2.1889%	0.615325	1.3469%	6.797260	0.091552	0.622301
15-Aug-02	2.1889%	0.593711	1.2996%	7.293151	0.094780	0.691247
17-Feb-03	2.1889%	0.572629	1.2534%	7.802740	0.097802	0.763125
15-Aug-03	2.1889%	0.552733	1.2099%	8.293151	0.100337	0.832113
16-Feb-04	2.1889%	0.533212	1.1672%	8.800000	0.102709	0.903843
16-Aug-04	2.1889%	0.514482	1.1262%	9.298630	0.104717	0.973724
15-Feb-05	2.1889%	0.496508	1.0868%	9.800000	0.106508	1.043774
15-Aug-05	2.1889%	0.479067	1.0486%	10.295890	0.107966	1.111609
15-Feb-06	2.1889%	0.462239	1.0118%	10.800000	0.109274	1.180164
15-Aug-06	2.1889%	0.446002	0.9763%	11.295890	0.110277	1.245678
15-Feb-07	2.1889%	0.430336	0.9420%	11.800000	0.111152	1.311593
15-Aug-07	2.1889%	0.415219	0.9089%	12.295890	0.111755	1.374123
15-Feb-08	2.1889%	0.400634	0.8770%	12.800000	0.112250	1.436798
15-Aug-08	2.1889%	0.386561	0.8461%	13.298630	0.112526	1.496442
16-Feb-09	2.1889%	0.372909	0.8163%	13.805479	0.112689	1.555727
17-Aug-09	2.1889%	0.359739	0.7874%	14.304110	0.112636	1.611152
15-Feb-10	2.1889%	0.347239	0.7601%	14.802740	0.112512	1.665486
16-Aug-10	2.1889%	0.334976	0.7332%	15.301370	0.112195	1.716730
15-Feb-11	2.1889%	0.323273	0.7076%	15.802740	0.111823	1.767104
15-Aug-11	2.1889%	0.311917	0.6828%	16.298630	0.111280	1.813718
15-Feb-12	2.1889%	0.300961	0.6588%	16.802740	0.110692	1.859936
15-Aug-12	2.1889%	0.290389	0.6356%	17.301370	0.109974	1.902695
15-Feb-13	2.1889%	0.280189	0.6133%	17.805479	0.109202	1.944401
15-Aug-13	2.1889%	0.270346	0.5918%	18.301370	0.108301	1.982056
17-Feb-14	2.1889%	0.260747	0.5708%	18.810959	0.107364	2.019618
15-Aug-14	2.1889%	0.251687	0.5509%	19.301370	0.106335	2.052417
16-Feb-15	41.1029%	0.242798	9.9797%	19.808219	1.976803	39.156944

The formula asks that we sum all of the products of PV × Time plus PV × Time^2, and then divide by $(1+\text{YTM}/2)^2$:

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$$\text{Convexity} = \frac{\dfrac{\sum_{t=1}^{n}\left(t^2 \times PVCF_t + t \times PVCF_t\right)}{\text{Dirty Price}}}{\left(1+\dfrac{YTM}{2}\right)^2}$$

Dollar convexity is calculated by multiplying the convexity by the market value of the bond(s):

$$\text{Dollar Convexity} = 84.934 \times \frac{104.5433}{100} = 88.79$$

This can be understood to mean the pricing error resulting from using modified duration to predict the new price of the bullet. It is best to think of it as a relative term.

Bullet Convexity
For the bullet:

					Convexity:	63.505676
Date	**Cash Flow**	**PV Factor**	**CF PV**	**Time**	**PV × T**	**PV × T^2**
2-May-95			104.5376%			
15-Aug-95	3.7500%	0.979899	3.6746%	0.287671	0.010571	0.003041
15-Feb-96	3.7500%	0.946570	3.5496%	0.791781	0.028105	0.022253
15-Aug-96	3.7500%	0.914241	3.4284%	1.290411	0.044240	0.057088
17-Feb-97	3.7500%	0.882677	3.3100%	1.800000	0.059581	0.107245
15-Aug-97	3.7500%	0.852857	3.1982%	2.290411	0.073252	0.167778
16-Feb-98	3.7500%	0.823571	3.0884%	2.797260	0.086390	0.241656
17-Aug-98	3.7500%	0.795290	2.9823%	3.295890	0.098295	0.323968
15-Feb-99	3.7500%	0.768422	2.8816%	3.794521	0.109342	0.414901
16-Aug-99	3.7500%	0.742035	2.7826%	4.293151	0.119463	0.512871
15-Feb-00	3.7500%	0.716829	2.6881%	4.794521	0.128882	0.617927
15-Aug-00	3.7500%	0.692346	2.5963%	5.293151	0.137426	0.727417
15-Feb-01	3.7500%	0.668700	2.5076%	5.797260	0.145374	0.842768
15-Aug-01	3.7500%	0.645861	2.4220%	6.293151	0.152419	0.959195
15-Feb-02	3.7500%	0.623802	2.3393%	6.797260	0.159006	1.080802
15-Aug-02	3.7500%	0.602497	2.2594%	7.293151	0.164779	1.201757
17-Feb-03	3.7500%	0.581696	2.1814%	7.802740	0.170206	1.328072
15-Aug-03	3.7500%	0.562044	2.1077%	8.293151	0.174792	1.449576
16-Feb-04	3.7500%	0.542744	2.0353%	8.800000	0.179106	1.576129
16-Aug-04	3.7500%	0.524207	1.9658%	9.298630	0.182790	1.699700
15-Feb-05	103.7500%	0.506401	52.5391%	9.800000	5.148828	50.458517

The dollar convexity for the bullet:

$$\text{Dollar Convexity} = 63.507 \times \frac{104.5433}{100} = 66.391$$

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We can now describe our two possible positions more accurately:

Bond	Maturity	Coupon	Dirty Price	YTM	Dollar Duration	Dollar Convexity
C	15-Feb-05	7 1/2 %	104.5433	7.0724%	7.1139	66.39
Dumbbell						
A	15-Feb-00	7 1/8 %	49.4122	6.8960%	1.9521	
B	15-Feb-15	11 1/4 %	55.1311	7.4255%	5.1618	
Total			104.5433	7.2802%	7.1139	88.79

Arbitrage Result Number 3
This result is far more interesting: We cannot only take a yield gain of 0.21% by buying the Dumbbell and selling Bond C, but we get a position with far greater convexity, which, as we have seen, is a desirable property.

Something doesn't add up here!

Horizon Yield

Our analysis so far has paid no attention to the "expected developments" of yields over time. Specifically, we are treating each cash flow as if it can be reinvested at the various IRRs we are using.

Horizon yield is similar to realized compound yield in that it wants to treat the reinvestment of each cash flow at a market rate.

The simplest case is to use market PV factors to calculate cash flow reinvestment rates through a future date. This makes use of the terms structure of yields as it stands currently.

For example, suppose we wish to think in terms of a horizon of about nine months. For simplicity's sake, let us use the date of 15 February 1996, since we can readily calculate all the relevant PV factors. We will use the "generic" Treasury pricing curve calculated off the STRIPS market, as it reflects the correct zero-coupon Treasury rates.

To restate the yield curve as of 15 Feb 96 is quite simple. The logic is that we can invest today's PV of any future cash flow at the market rate existing today for maturity on 15 Feb 96.

For example, $1 invested today through 15 Feb 96 will be worth:

$$\frac{1}{0.952344} = 1.050041$$

$1 receivable on 15 August 1995 has a market PV today of 0.983125. Invested through 15 Feb 96 it will be worth:

$$\frac{0.983125}{0.952344} = 1.032322$$

Following the same logic, $1 receivable on 15 Feb 2000 is worth on 15 Feb 96:

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$$\frac{0.722656}{0.952344} = 0.758819$$

We can thus calculate the PV as of 15 Feb 96 of every cash flow in both the dumbbell and the bullet:

Date	PV Factor	Bullet CF	Dumbbell CF
2-May-95	1.050041	109.6733%	109.8843%
15-Aug-95	1.032322	3.8712%	4.0342%
15-Feb-96	1.000000	3.7500%	3.9079%
15-Aug-96	0.970468	3.6393%	3.7925%
17-Feb-97	0.936833	3.5131%	3.6611%
15-Aug-97	0.903527	3.3882%	3.5309%
16-Feb-98	0.872847	3.2732%	3.4110%
17-Aug-98	0.842494	3.1594%	3.2924%
15-Feb-99	0.813782	3.0517%	3.1802%
16-Aug-99	0.786710	2.9502%	3.0744%
15-Feb-00	0.758819	2.8456%	39.5808%
15-Aug-00	0.731911	2.7447%	1.6021%
15-Feb-01	0.706809	2.6505%	1.5471%
15-Aug-01	0.681870	2.5570%	1.4926%
15-Feb-02	0.657588	2.4660%	1.4394%
15-Aug-02	0.633962	2.3774%	1.3877%
17-Feb-03	0.610500	2.2894%	1.3363%
15-Aug-03	0.588679	2.2075%	1.2886%
16-Feb-04	0.567022	2.1263%	1.2412%
16-Aug-04	0.547334	2.0525%	1.1981%
15-Feb-05	0.527810	54.7603%	1.1553%
15-Aug-05	0.502871		1.1007%
15-Feb-06	0.483183		1.0576%
15-Aug-06	0.465135		1.0181%
15-Feb-07	0.446267		0.9768%
15-Aug-07	0.429040		0.9391%
15-Feb-08	0.412305		0.9025%
15-Aug-08	0.396719		0.8684%
16-Feb-09	0.380640		0.8332%
17-Aug-09	0.366366		0.8019%
15-Feb-10	0.351928		0.7703%
16-Aug-10	0.338146		0.7402%
15-Feb-11	0.324856		0.7111%
15-Aug-11	0.312059		0.6831%
15-Feb-12	0.299918		0.6565%
15-Aug-12	0.288433		0.6314%
15-Feb-13	0.276948		0.6062%
15-Aug-13	0.266120		0.5825%
17-Feb-14	0.255947		0.5602%
15-Aug-14	0.246267		0.5391%
16-Feb-15	0.237244		9.7514%

The simple horizon yield through 15 Feb 96 is thus based on both strategies' market value today compared to their future values on the horizon date:

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$$\text{Bullet} = \frac{109.6733\%}{104.5433\%} - 1 = 4.9127\% \qquad \text{Dumbbell} = \frac{109.8843\%}{104.5433\%} - 1 = 5.1088\%$$

Once again, the dumbbell proves to be the "richer" strategy.

HORIZON VALUE AND VOLATILITY

The resolution of this arbitrage lies in modeling future developments in the yield curve to incorporate interest rate volatility.

Specifically, we need to use a binomial tree to model either the short-term rate (a ***single factor model***) or the entire term structure of forward rates (a ***multiple factor model***).

Building binomial trees is beyond the scope of this module, but we can summarize the results, using a simplified single-factor tree and a simple dumbbell-bullet comparison[1].

Bond	Maturity	Coupon	Price	Per Cents	YTM	Duration	Convexity
C	20 years	10%	100		10%	8.5795	121.5637
Dumbbell							
A	10 years	10%	100	27.37%	10%	6.2311	
B	30 years	10%	100	72.63%	10%	9.4646	
Total			100		10%	8.5795	133.8108

The dollar duration weighted YTM of the dumbbell is identical to that of the bullet, indicating no arbitrage opportunity exists.

The horizon yield for both is also the same, 10% over a five-year horizon. This is because the yield curve is flat at 10%.

The convexities of the two positions are not the same, however. The greater convexity of the dumbbell appears attractive given that all else is equal.

If we make some simplifying assumptions, we can test the two positions for assumed changes in the level of the 6-month rate over a 5-year horizon period:

1. Changes occur today and instantaneously affect both reinvestment rates (prior to 5 years) and horizon rates (after 5 years).

2. All instantaneous yield curve shifts occur in parallel fashion across the entire term structure.

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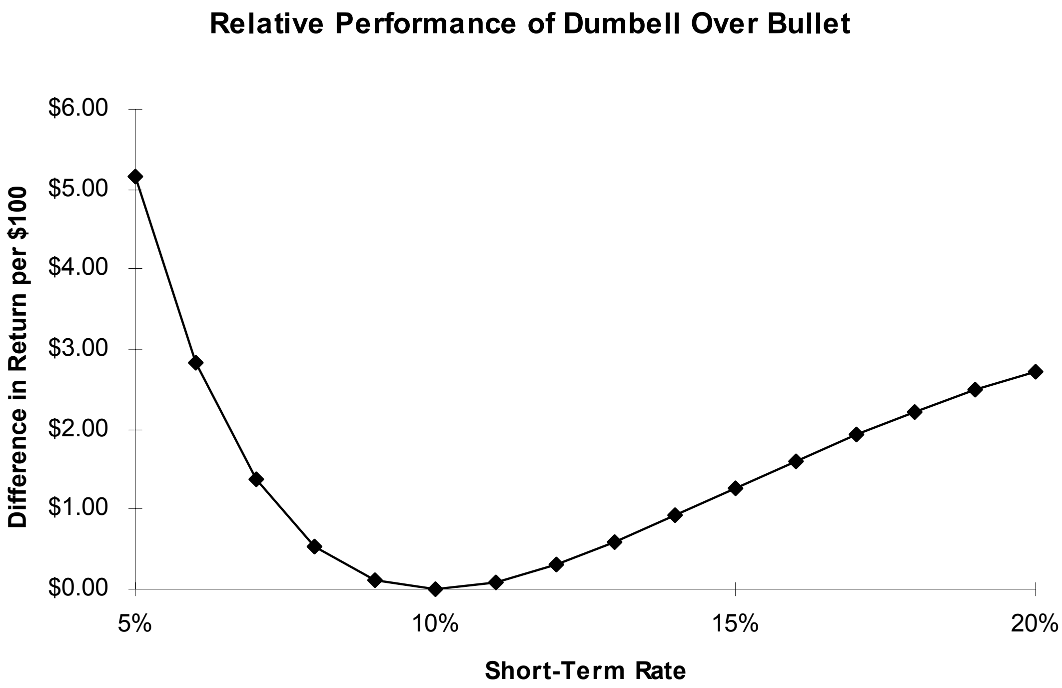
[1] The following example is taken from *Active Total Return Management of Fixed Income Portfolios*, Dattatreya and Fabozzi; Probus Publishing, Chicago, 1989.

By comparing the horizon value of the two strategies to see which is worth more after five years, we can observe the effects of the convexity differences:



This suggests that no matter what happens, the added convexity of the dumbbell is worth owning.

But it does not tell the full story.

Our simple assumptions bear no resemblance to the way interest rates are likely to develop.

Rates do not instantaneously move by massive amounts and then hold still for 5 years.

Furthermore, we make no discrimination about how far rates are **likely** to move.

To incorporate a more realistic projection of horizon return, we need to use volatility and a binomial tree to measure the "likely" horizon values in terms of probability.

The graph which follows shows the excess value of the dumbbell strategy at various possible levels of the short-term rate 5 years from now. Clearly it is not always profitable to hold the dumbbell and sell the bullet.

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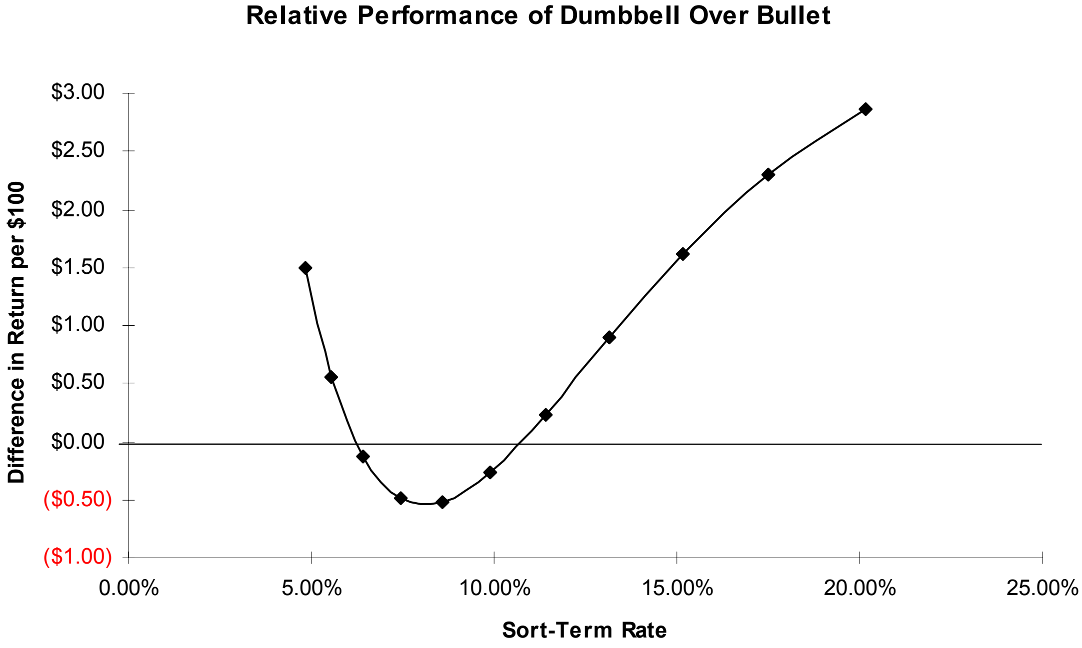

Dattatreya and Fabozzi offer the following values for the horizon value of the two strategies, weighted by volatility and probability:

Short Rate	Probability	Bullet FV	Dumbbell FV	Difference
4.81%	0.1%	$208.36	$209.86	$1.50
5.56%	1.0%	$199.36	$199.92	$0.55
6.43%	4.4%	$190.07	$189.94	($0.13)
7.42%	11.7%	$180.60	$180.12	($0.48)
8.56%	20.5%	$171.11	$170.60	($0.51)
9.88%	24.6%	$161.79	$161.54	($0.26)
11.40%	20.5%	$152.84	$153.07	$0.24
13.15%	11.7%	$144.43	$145.32	$0.89
15.17%	4.4%	$136.76	$138.37	$1.61
17.49%	1.0%	$129.99	$132.28	$2.30
20.16%	0.1%	$124.25	$127.11	$2.87
Probability-Weighted Returns		**$162.24**	**$162.27**	**$0.03**

Finally, we can resolve this apparent market inefficiency. On a probability-weighted basis both strategies have the same expected value after 5 years.

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Relative Value Concepts

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